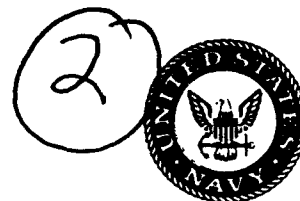


Naval Research Laboratory

Washington, DC 20375-5320



AD-A254 353



NRL/MR/6542-92-7101

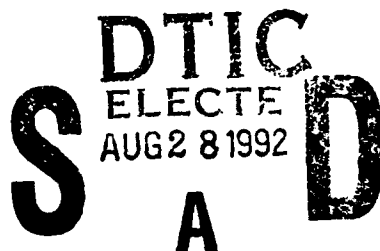
Electron Terms and Radiation from Crystals

K. H. HELLWEGE (author)

W. L. FAUST (translator)

*Laser Physics Branch
Optical Sciences Division*

August 13, 1992



Translation and Commentary on a Classical (1948) Paper from the German



Approved for public release; distribution unlimited.

92 8 27 045

REPORT DOCUMENTATION PAGE			Form Approved OMB No 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE August 13, 1992		3. REPORT TYPE AND DATES COVERED Interim
4. TITLE AND SUBTITLE Electron Terms and Radiation from Crystals			5. FUNDING NUMBERS	
6. AUTHOR(S) K. H. Hellwege (author) W. L. Faust (translator)				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Research Laboratory 4555 Overlook Ave. Washington, DC 20375-5000			8. PERFORMING ORGANIZATION REPORT NUMBER NRL/MR/6542-92-7101	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSORING / MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES Translation and Commentary on Classical (1948) Paper from the German				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) This paper describes the effect of a crystal field, according to site symmetry, upon the magnetic quantum-level structure of an atomic ion, as expressed in electric dipole transitions (a corresponding treatment for magnetic dipole transitions, in the original German, is not included).				
14. SUBJECT TERMS Crystal field Atomic ion Magnetic quantum numbers			15. NUMBER OF PAGES 43	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

K. H. Hellwege, Annalen der Physik 4, 95 (1948)
 "Electron Terms and Radiation from Atoms in Crystals"

[There is a slight irregularity of pagination between this and the prior article.]

TABLE OF CONTENTS

The original pagination is preserved, for convenient comparison.

Translator's Explanatory Remarks, Title, By-Line, Journal Citation.....	95
Author's Overview.....	95
I. Introduction.....	95
1. Overview of the Problem.....	95
2. Method.....	96
3. The States of a Free Atom.....	97
II. States in the Crystal.....	100
4. Cyclic Classes with $p=1,2,3,4,6$	100
5. Cyclic-Inverse Classes.....	101
6. Cyclic Classes with Inversion Centers.....	102
7. Cyclic Classes with Vertical Mirror Planes.....	102
8. Dihedral Classes.....	106
9. Cyclic-Inverse Classes with Reflection Planes.....	106
10. Dihedral Classes with Inversion Centers.....	106
11. Tabular Summation.....	106
A block of pages, 107 through mid-page 121... comprising Tables 2 and 3, has been transported entire from its position within the original German text, to an Appendix (see note following p. 106).	
III Selection Rules for Electric Dipole Radiation.....	121
12. Cyclic Classes.....	121
13. Cyclic-Inverse Classes.....	122
14. Cyclic Classes with Inversion Centers.....	123
15. Cyclic Classes with Vertical Mirror Planes.....	123
16. Dihedral Classes.....	124
17. Cyclic-Inverse Classes with Reflection Planes.....	125
18. Dihedral Classes with Symmetry Center.....	125
IV. Domain of Application of the Theory.....	125
Editor's note on date of receipt of manuscript.....	126
Footnotes, References	
Appendix (containing tabular data, the bulk of the results)...pp.	107-121
Errata: translated text and corrected Tables.....pp.	357-358

DTIC QUALITY INSPECTED 5

Accession For	
NTIS	CRA&I
DTIC	TAB
Unannounced	
Justification	
By	
Distribution/	
Availability	
Dist	Availability
A-1	Spec

TRANSLATION FROM THE GERMAN BY WALTER L. FAUST, FROM THE GERMAN ORIGINAL, WITH ADDITION OF SOME EXPOSITORY REMARKS; THE LATTER APPEAR IN SQUARE BRACKETS, AS [bold...].

AUTHORIZATION FOR PUBLICATION HAS BEEN GRANTED BY THE PUBLISHER AND BY DR. HELLWEGE, PRESENTLY PROFESSOR EMERITUS IN THE FACULTY OF MATHEMATICS AND PHYSICS AT THE TECHNISCHE HOCHSCHULE DARMSTADT. PROF. HELLWEGE POINTED OUT THAT THE ORIGINAL PAPER WAS "NOT FREE FROM ERRORS". THESE WERE RECTIFIED IN ERRATA PUBLISHED ON P. 357-8 IN THE SAME VOLUME AS THE ORIGINAL PAPER. THE ERRATUM HAS BEEN GIVEN A PARALLEL TREATMENT, AND INCLUDED IN AN APPENDIX CONTAINING TABLES 2 AND 3 FROM THE INITIAL PAPER. FURTHER, THE ERRONEOUS PARTS OF THE ORIGINAL, PERTAINING TO D_{2s} , D_{3s} , AND D_6 (${}^2\Gamma_5$ ONLY) HAVE BEEN EARMARKED AS SUCH IN TEXT.

THE ORIGINAL PAGINATION IS PRESERVED, FOR CONVENIENT COMPARISON.

K. H. Hellwege, *Annalen der Physik* 4, 95 (1948)

"Electron Terms and Radiation from Atoms in Crystals"

[There is a slight irregularity of pagination between this and the prior article.]

I. Term Splitting and Electric Dipole Radiation

[The original volume included a following section "II. Magnetic Dipole Radiation"; this has not been included here.]

Overview of Contents

For all 27 noncubic symmetry classes the following are determined:
a) the electron function, in the zero-order approximation, and the accompanying crystal quantum numbers; b) the number and degeneracy of the components into which one term of the supposed angular momentum quantum number of a free atom splits; c) selection rules for the crystal quantum numbers, and polarization of the emission for electric-dipole radiation.

I. Introduction

1. Overview of the Problem

If one constructs within a crystal an atom or ion, at the outset undistorted by any external influence, then one can in first approximation trace back the perturbation of its state to an electromagnetic field (microfield) generated by the lattice neighbors at the position of the atom. Through this the spectral lines become split into a series of components, which are polarized in a characteristic way with respect to the preferred direction of the microfield. Since knowledge of the symmetry of the perturbation potential alone already permits one to give the greatest number of components which may appear (and further, also upon perturbation, the least degeneracy still available for individual components, into which a degenerate term of the unperturbed atom splits because of the perturbation), it approaches theoretical determination of the term splitting in a crystal... in that one identifies the symmetry of the perturbation potential with the point symmetry of the atom contemplated, therefore with one of the 32 symmetry classes. This is carried out with the help of the group theory of Bethe¹ and Opechowsky².

In the following the same problem is freshly treated, avoiding group theoretical methods. The process, which incidentally was used in a special case partially prepared by Bethe³ for treatment of the Zeeman effect in crystals, allows one to determine from it the electron state in the crystal in zeroth approximation and to classify the transformation behavior under covering operations of the lattice through "crystal quantum-numbers"; that is, to determine the symmetry character in the sense of Hund⁴.

page 96

The enumeration of the terms follows then very simply from the symmetry properties of the eigenfunctions. At the same time there follows the treatment of systems with half and whole number angular momentum, both from the same overlying analytic hypothesis; and the difficulties already noted by Opechowski² for odd number values, which are to be overcome in group theory, don't even appear at the outset.

The results for the 27 noncubic crystal classes are, for convenience, presented together tabularly for reference⁵.

Afterwards the selection rules due to symmetry are given for transitions tied to electric dipole radiation⁶, with help from the crystal quantum numbers. Thus one obtains the polarization of spectral lines relative to the preferred direction of the microfield. These comply with the preferred directions of the total crystal, to which the radiation and oscillation directions of light naturally are oriented in an experiment. Thus one can determine the crystal and angular momentum quantum numbers of the terms from the number of lines and polarization-- and the symmetry class of the microfield, not absolutely matched with the symmetry class of the total crystal⁷. The capability thus won, essential for the experimenter, to analyze crystal spectra, should justify this new work on these problems.

2. Method

If one writes the H-operator of an atom with N spin-electrons in the crystal in the form

$$H = H_0 + K, \quad (1)$$

wherein H_0 is the operator of the free atom and K signifies the operator of the crystal field, then H_0 is invariant under all rotations of the coordinate system (origin in the atom nucleus) about the origin, under inversion in the origin, and under reflection in all planes through the origin. K and thus H , in contrast, are invariant only under the finite selection of rotations and reflections which are contained in the point symmetry of the atom contemplated, in the crystal lattice.

If the state of the free atom is designated by ψ_{JM} , that is

$$H_0 \psi_{JM} = W_J \psi_{JM}, \quad (2)$$

where states distinguished only by the value of the magnetic quantum number M

$$M = J, J-1, \dots, -J \quad (3)$$

belong to the same eigenvalue W_J , then there are likewise enough states u_i satisfying

$$H u_i = (H_0 + K) u_i = W_i \quad (1')$$

in the crystal, linear combinations

$$u_i = \sum_{J,M} \psi_{JM} a_{JM,i} \quad (4)$$

of all ψ_{JM} .

According to a familiar theorem⁸, states which belong to a given eigenvalue W_i (degenerate with one another) transform linearly into one another under a coordinate transformation under which the H-operator is invariant. A state therefore belongs to a simple ["einfach"= singlefold, nondegenerate] eigenvalue within the crystal, if it goes into itself up to a constant factor, the norm of which is of measure 1, under all invariance transformations. A state belongs to a twofold eigenvalue in the crystal, if it yields a second state, linearly independent of it, so that the two transform into one another - and so forth.

In order to learn whether nondegenerate terms occur in a crystal, one has therefore only to consider whether states u_i allow themselves to be constructed in accordance with Eq. (4), which transform into themselves, up to a factor, under all symmetry operations of the point symmetry-- and correspondingly for the case of multiple terms.

Since accordingly only the symmetry, but not the strength of the crystal field is of interest, it suffices to consider the crystal state in the zeroth approximation; that is to let the crystal field vanish under fixed symmetry. Then u_i reduces to a linear combination only of the $2J+1$ states of the free atom, degenerate with one another, of the eigenvalue W_j , that is Eq. 4 goes over into

$$u_i = \sum_M \psi_M a_{M,i}, \quad (J \text{ fixed}). \quad (5)$$

The general result of the investigation has been known since Bethe¹: Corresponding to the lower symmetry in the crystal, the degeneracy of the terms remains no longer $(2J+1)$ -fold, as in the free atom; yet in many crystals the symmetry is sufficiently high to force degeneracies. Thus, in these cases a term of the atom splits up in the crystal into less than $2J+1$ components.

Now, for an overall view how the states built from the ψ_M transform in the crystal, in the zeroth approximation, the transformation properties of the ψ_M are themselves next assembled.

Here one can use the fact that the ψ_M always can be selected so that under rotations they transform like monomials⁸

$$\psi_{JM} = e^{iM\phi} \begin{pmatrix} 2J \\ J+M \end{pmatrix}^{1/2} \xi^{J+M} \eta^{J-M}, \quad \text{where } \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

But this correlation does not extend itself further to reflections. We therefore go back directly to the states of single electrons.

3. The States of a Free Atom

An atom with a spin electron has according to Pauli⁹ the states

$$\phi_{1m} = R_{n1}(r) (2l+1)^{1/2} \{ (l+m+1/2)^{1/2} Y(\theta\phi)_{l,m-1/2} \xi(s) - (l-m+1/2)^{1/2} Y(\theta\phi)_{l,m+1/2} \eta(s) \} \quad (6) \\ \text{if } j = l + 1/2,$$

$$\phi_{2m} = R_{n1}(r) (2l+1)^{1/2} \{ (l-m+1/2)^{1/2} Y(\theta\phi)_{l,m-1/2} \xi(s) + (l+m+1/2)^{1/2} Y(\theta\phi)_{l,m+1/2} \eta(s) \} \\ \text{if } j = l - 1/2.$$

In this l is the orbital momentum, j the total rotational momentum; $m=m_j$ measures the z -component of the total rotational momentum. The Y_{lm} are the spherical surface functions¹⁰, $\xi(s)$ and $\eta(s)$ the Pauli spin functions.

If one neglects initially, for an atom with N spin electrons, their interactions with one another, then its states are simple products of states of the form⁶:

$$\psi_M = \prod_{k=1}^N R(r) \{c Y(\theta\phi)_{1,m-\frac{1}{2}} \xi + d Y(\theta\phi)_{1,m+\frac{1}{2}} \eta(s)\} \quad (7)$$

[Here each variable and each constant should carry the index k, with the exception of s.]

where the c_k and d_k are the constants formed from the quantum numbers from ref. 6 and for simplicity m is written instead of m_j . Each such product is thereby characterized, in that it has a total rotational momentum with z-component

$$M \hbar = \sum_{k=1}^N m_k \hbar. \quad (8)$$

All products with equal R_k , l_k , and j_k of individual electrons, but distinct m_k are of course degenerate with one another. If one imagines the mutual perturbation of the electrons switched on, then the true states follow, linear combinations of products⁷ which are distinguished only by the m_k . There also, after turning on the interaction, the z-components of the rotational momentum remain quantized; only such products⁷ get linearly combined as define the same M. It is one of the numbers M^3 ; and J measures the total rotational momentum appearing upon coupling the electrons. To obtain the ultimate state, there are now still to be carried out all possible permutations of the electrons; and the functions so obtained are to be combined linearly.

Now for the following we are interested in all the behavior of the state so obtained under a certain selection of special rotations and reflections of the coordinate system; and indeed it can easily be shown that it behaves exactly like the product⁷ from which it emerged. For proof, there is first to be understood the path from the (x,y,z)-system to the (x',y',z')-system, under the rotation (α, β, γ) of the coordinate system about the Euler angles α, β, γ , to follow: First it is rotated around the angle $-\pi \leq \alpha \leq \pi$ about the positive z-axis, then around the angle $-\pi \leq \beta \leq \pi$ [error in German text: β was given as θ] about the new positive y-axis so produced, and finally again around the angle $-\pi \leq \gamma \leq \pi$ about the new positive z-axis produced through both the first rotations. The transformation of the spherical functions occurring in Eq. 6 is easily written. The spin functions ξ and η transform, as well, in accordance with¹¹ #.

$$\begin{aligned} \xi(s) &= \cos(\beta/2) e^{i\frac{1}{2}(\alpha+\gamma)} \xi'(s) - \sin(\beta/2) e^{i\frac{1}{2}(\alpha-\gamma)} \eta'(s) \\ \eta(s) &= \sin(\beta/2) e^{-i\frac{1}{2}(\alpha-\gamma)} \xi'(s) + \cos(\beta/2) e^{-i\frac{1}{2}(\alpha+\gamma)} \eta'(s). \end{aligned} \quad (9)$$

The reflection in the origin of coordinates (inversion) corresponds to the transformation

$$\begin{aligned} \xi &= \xi' \\ \eta &= \eta'. \end{aligned} \quad (10)$$

The fact that ξ and η remain unchanged¹² depends upon the fact that rotation of the positive x-axis into the positive y-axis after reflection occurs as before, in the same rotational sense.

*[Translator's commentary: The expression is a special case of the general one for the coefficients coupling arbitrary j, m and j, m' , and α, β, γ ; the latter is developed by M. E. Rose, Elementary Theory of Angular Momentum (Wiley, NY 1957), p.52, Eq. 4.13.]

With that the transformation of the product (7) can be written without further ado, in the following special cases:

1. Rotation through α about the z-axis, that is rotation $(\alpha 0 0)$:

$$\psi_M(r\theta\phi) = \psi_M(r', \theta', \phi' + \alpha) = e^{iM\alpha} \psi_M(r' \theta' \phi'). \quad (11)$$

2. Rotation through π about the y-axis, that is rotation $(0 \pi 0)$

$$\begin{aligned} \psi_M(r\theta\phi) &= \psi_M(r', \pi - \theta', \pi - \phi') = \exp(iM - \sum_k j_k) \pi \psi_{-M}(r' \theta' \phi') \\ &= (-1)^{M - \sum j} \psi_{-M}(r' \theta' \phi'). \end{aligned} \quad (12)$$

[where $\sum j = \sum_k j_k$.]

There ψ_{-M} is distinguished from ψ_M only in that, in the product (7), that is in all the individual functions (6) the signs of all m_j are reversed, that is ψ_M and ψ_{-M} have opposite rotational senses of the total rotational momentum around the z-axis#.

3. Inversion: $\psi_M(xyz) = \psi_M(-x', -y', -z') = (-1)^{\sum l} \psi_M(x' y' z')$. (13)

States with even $\sum l = \sum_k l_k$ are called even; states with odd $\sum l$ are called odd.

4. p fold rotinversion, that is rotation $(2\pi/p, 0, 0)$ and inversion: It is

$$\begin{aligned} \psi_M(r\theta\phi) &= \psi_M(r', \pi - \theta', \phi' + 2\pi/p - \pi) = (-1)^{\sum l} \exp(iM 2\pi/p) \psi_M(r' \theta' \phi'), \\ &= \exp(i 2\pi/p (M + p/2 \sum l)) \psi_M(r' \theta' \phi') = \exp(i M_1 2\pi) \psi_M(r' \theta' \phi'), \end{aligned} \quad (14)$$

[strikeout per erratum]

where ~~M is an arbitrary whole number~~ [The context defines M_1].

5. Reflection in the zx-plane; that is rotation $(0 \pi 0)$ [for which see 2. above] and inversion: It is according to (12) and (13)

$$\begin{aligned} \psi_M(xyz) &= \psi_M(x', -y', z') = \exp(i\pi(M - \sum j)) (-1)^{\sum l} \psi_{-M}(x' y' z') \\ &= (-1)^{M + \sum(1-j)} \psi_{-M}(x' y' z'). \end{aligned} \quad (15)$$

[Where $\sum(1-j) = \sum_k (l_k - j_k)$.]

The constant factors appearing in these formulas have both the following properties:

a) They are the same for all products (7) which appear in a linear combination of such products, since the products are distinguished neither in M nor in the l_k and j_k ¹³.

b) They are independent of permutations of the electrons, since they always are summed over all electrons.

From this it follows that Eq. (11) to (15) already give the transformation behavior of the ultimate states of the free atom. Therefore, in future¹⁴ ψ_M will not designate the products (7), but rather the true states of the free atom built from them. From these ψ_M , then, the zeroth approximation states within the crystal are linearly assembled according to Eq. (5).

#[Translator's commentary: The ϕ -dependence of ψ is $\exp(iM\phi)$. This accounts for Eq. (11) immediately. In Eq. 12, we have $\theta \rightarrow \pi - \theta$, $\phi \rightarrow \pi - \phi$. Consider first $\phi \rightarrow \pi - \phi$: Rotation of ϕ through π gives a multiplicative factor of $\exp(iM\pi)$; this is like Eq. (11) but with a rotation of π . $\phi \rightarrow -\phi$ takes ψ_M into ψ_{-M} . $\theta \rightarrow \pi - \theta$ takes $\cos\theta$ into $-\cos\theta$. The even/odd Legendre polynomials are functions of $\cos\theta$ even/odd in the argument, accounting for the j_k term when j_k is integral. For half-integral j_k : $\theta \rightarrow \pi - \theta$ is equivalent to inversion through

II. States in the Crystal

First, in Table 1 the 27 symmetry classes investigated are arranged in seven columns. At the head of each column the symmetry elements are given, through which the symmetry classes of the column are characterized. In each line stand classes with equal numerical order p of the principal axis.

Table 1

column→ p↓	1	2	3	4	5	6	7
	A_p	J_p	A_p+J	$A_p+E_{ }$	$A_p+A_{2\perp}$	$J_p+E_{ }$	A_p+A_2+J
odd 1	C_1	C_1					
3	C_3	C_{3i}		C_{3v}	D_3		D_{3d}
even 2	C_2	C_2	C_{2h}	C_{2v}	D_2		D_{2h}
4	C_4	S_4	C_{4h}	C_{4v}	D_4	D_{2d}	D_{4h}
6	C_6	C_{3h}	C_{6h}	C_{6v}	D_6	D_{3h}	D_{6h}

A_p = p -fold covering axis; J_p = p -fold inversion axis; J =inversion center
 E =reflection plane; $||$ =parallel to p -fold principal axis; \perp =perpendicular...

This arrangement, like each alternative one, is not free of arbitrariness. It is for example possible to identify many of the classes cited in column 2 instead through an inversion axis through other symmetry elements, for example C_4 through a mirror plane. The arrangement of Table 1 is chosen because it is especially well suited to the mathematical treatment of problems.

4. Cyclic Classes C_p with $p=1,2,3,4,6$ [Column 1 of Table 1]

The perturbing potential behaves as a single symmetry element, a p -fold covering axis [floor axis] which may coincide with the z -axis. Of nondegenerate states in the crystal⁵, it must therefore be required that, under rotation of the coordinate system through the angle $2\pi/p$ about the z -axis, they are multiplied by a factor of modulus 1, that is it is required

$$u_i(r, \theta, \phi + 2\pi/p) = D_i(z) \cdot u_i(r, \theta, \phi). \quad (16)$$

That is only possible if all ψ_M belonging to function (5) are multiplied with the same factor $D(z)$; that is, according to Eq. (11), if only such ψ_M belong to the function, as have M -values distinguished according to multiples of p [classification by mod p]. Therefore

$$M = \mu \pmod{p} \text{ holds for them.} \quad (17)$$

At the same time, for whole-number M , that is for even electron number N , the "crystal quantum number" μ assigned to the principal crystallographic axis is one of the numbers

$$\mu = 0, 1, 2, 3, \dots, p-1, \quad (18)$$

or, which is the same,

$$\mu = 0, \pm 1, \dots, \pm [p/2] \quad ^{15}. \quad (18')$$

If on the other hand the electron number N is odd, then the M -value is half-integral; and it gives for μ the possibilities

$$\mu = 1/2, 3/2, \dots, (2p-1)/2 \quad (19)$$

Continued commentary:

the origin plus $\phi \rightarrow \pi - \phi$. The former is an identity operation (see bottom of p. 98, "... remain unchanged"), and the latter contributes a factor of $i = \exp(i\pi/2)$.

page 101

or, which is the same thing,

$$\mu = \pm 1/2, \pm 3/2, \dots, \pm((p-1)/2 + 1/2). \quad (19')$$

All states assembled in this wise are multiplied actually by the factor

$$D(z) = \exp(i\mu 2\pi/p). \quad (20)$$

That is, there are, in accordance with (18) and (19), exactly p distinct families of functions, which are characterized with respect to the value of the rotational factor $D(z)$ by their μ -values. Since, only naturally, each of the M -values accompanying the same J belongs to one of the μ -values, it is always possible to sort out the ψ_M according to the accompanying μ ; and altogether $2J+1$ new functions can be formed, of which each belongs to one of the p through the characteristic μ . Since each of these states is nondegenerate in the crystal [for distinct states of a common μ , the curvature of the wave function will progress], this means: Each $(2J+1)$ -fold degenerate term of the free atom, under the influence of a perturbation potential of only cyclic symmetry, splits up completely into $2J+1$ components.

The number of terms belonging to the family μ is therefore as great as the number of those M -values in the row (3) which satisfy condition (17); that is, as one easily checks, equal to [see again note 15, re square brackets] Eq. (21)...

$$z_\mu = 1 + [(J+\mu)/p] + [(J-\mu)/p] = z_{-\mu}. \quad (21)$$

On this matter, it must also yet be pointed out that the case of a homogeneous magnetic field parallel to the z -axis belongs to the cyclic classes, as the limit case $p = \infty$. Eq. (17) becomes in the limit

$$\mu = M, \quad (22)$$

that is, here again there are p -- i.e. infinitely many-- families of states, which are characterized simply through the magnetic quantum number M . The states u_i are the ψ_M themselves, we have

$$z_M = z_{-M} = 1 \quad (23)$$

5. Cyclic-Inverse Classes, Column 2 of Table 1 [rotation and inversion compulsorily associated]

Of nondegenerate states in the crystal it is required:

$$u_i(r, \pi-\theta, \phi + \{2\pi/p\} - \pi) = D_i(z) u_i(r\theta\phi). \quad (24)$$

With that it develops, according to Eq. (14), that only such M_i combine, as have the same value for

$$D_i(z) = \exp(i\{2\pi/p\}(M_i \pm p/2 \Sigma l_i)) = I \cdot \exp(iM_i 2\pi/p) \quad (25)$$

that is for which

$$M_i \pm p/2 \Sigma l_i = \mu_i \pmod{p} \quad (26)$$

$$D_i(z) = \exp(i\mu_i 2\pi/p) = I \cdot \exp(i\mu 2\pi/p). \quad (27)$$

The possible values of μ_i are those of Eq. (18'), (19') with μ_i instead of μ . Thus μ_i , together with $M_i \pm (p/2) \Sigma l_i$, is whole or half integral; that is there can occur (in contrast with μ) half-integral values for even electron number and whole number values for odd electron number.

page 102

In the zeroth approximation which we use, where all the ψ_{μ} occurring in u , have the same I , it is possible to use I and μ instead of μ_1 in the sense of Eq. (27). However, it is often useful also to label the actual crystal states by this information. For example, the number of terms belonging to a given value of μ_1 is determined in this way: one first determines the number of terms belonging to a given μ with Eq. (21), and then goes over to μ_1 according to

$$\mu_1 = \mu \pm (p/2) \sum_k l_k \pmod{p} \quad (28)$$

(see Tables 2 and 3).

6. Cyclic Classes with Inversion Centers, Column 3 Table 1 [rotate and/or invert]

Besides the p -fold axis there enters the inversion center as a new symmetry element. Therefore, besides (17) there is required of nondegenerate states in the crystal

$$\mu_1(-x, -y, -z) = I_1 \cdot u_1(x, y, z). \quad (29)$$

However, this requirement is fulfilled by all states of the cyclic classes, since all ψ_{μ} belonging to the same J also have the same value of the sum $\sum_k l_k$, -- already, according to Eq. (13); and indeed with the same

$$I_1 = (-1)^{\sum l_k}, \quad (30)$$

[where again $\sum l_k$ means $\sum_k l_k$.]

The states are therefore labeled through the values of μI . States with $I=1$ are called even, and with $I=-1$ odd. All terms are nondegenerate.

7. Cyclic Classes with Vertical Mirror Planes, Column 4 from Table 1 [rotate and/or reflect]

Here there appears, besides the p -fold covering axis in the z -direction, a mirror plane passing through it, in which the zx -plane appropriately is placed¹⁶. Therefore, there appears, besides the requirement (17) for rotation, the requirement

$$u_1(x, -y, z) = S_1 \cdot u_1(xyz) \quad (31)$$

for reflection, which must be fulfilled additionally, for nondegenerate states. The states of the cyclic classes do not in general satisfy the requirement (31), since according to Eq. (15) the reflection of each μ_{μ} , instead of going into itself up to a factor goes into $\mu_{-\mu}$ up to a factor; and the cyclic states generally do not contain ψ_{μ} and $\psi_{-\mu}$ together. However, the correct states must fulfill the following conditions:

1. $D(z)$ must be defined on the basis of (16), that is only ψ_{μ} are combined which belong to the same μ (to the same family in regard to rotation).
2. S must be defined on the basis of (31), that is the states must
 - a) contain $\psi_{-\mu}$ as well as ψ_{μ} , and
 - b) they must be combined in such a way that all M -values occurring in a state yield the same factor S .

The condition 2a) first requires according to (17), that the following hold simultaneously:

$$M = \mu \pmod{p} \quad (32)$$

$$-M = \mu \pmod{p},$$

and therefore

$$2\mu = 0 \pmod{p} \quad (33)$$

which in general can be fulfilled for

$$\mu = 0 \text{ and } \mu = p/2. \quad (34)$$

From these possibilities, on the basis of (18) and (10), only the following cases actually can occur:

$$\mu = \begin{cases} 3/2 \text{ for odd } p = 3, \text{ for odd } N \\ - \text{ for even } p = 2, 4, 6, \text{ for odd } N \\ 0 \text{ for odd } p = 3, \text{ for even } N \\ 0, p/2 \text{ for even } p = 2, 4, 6, \text{ for even } N. \end{cases} \quad (35)$$

States with $\mu \neq 0, p/2$ are therefore necessarily degenerate; for example, in crystals with even numerical order [refers to the rotation axis; thus $p/2$ is integral], states of atoms with odd electron number are all degenerate.

For the nondegenerate states it is further required, as a result of condition 2b), that the sums of binomials

$$a_M \psi_M + a_{-M} \psi_{-M}, \quad (36)$$

which are all multiplied by the same factor S under reflection are:

$$a_M \psi_M(x-yz) + a_{-M} \psi_{-M}(x-yz) = S(a_M \psi_M(xyz) + a_{-M} \psi_{-M}(xyz)). \quad (37)$$

Now, however, according to Eq. (15)

$$\begin{aligned} & a_M \psi_M(x-yz) + a_{-M} \psi_{-M}(x-yz) \\ & = f \{ a_M \exp iM\pi \cdot \psi_{-M}(xyz) + a_{-M} \exp -iM\pi \cdot \psi_M(xyz) \} \end{aligned} \quad (38)$$

with

$$f = (-1)^{\sum j} \cdot \exp i\pi \sum j. \quad (39)$$

Equating coefficients with (37) gives the two equations

$$\begin{aligned} S \cdot a_M &= f \cdot \exp(-iM\pi) \cdot a_{-M} \\ S \cdot a_{-M} &= f \cdot \exp(iM\pi) \cdot a_M \end{aligned} \quad (40)$$

for the two unknowns S and a_M/a_{-M} , for which there result

$$\begin{aligned} S &= \pm f \\ a_{-M} &= \pm \exp iM\pi \cdot a_M \end{aligned} \quad (41)$$

with either the upper or the lower sign. Therefore, in nondegenerate states, only binomials

$$a_M (\psi_M \pm \exp iM\pi \psi_{-M}) \quad (42)$$

occur, which upon reflection are multiplied by the same factor $S = \pm f$. At the same time, in the case of even electron number $\exp(iM\pi)$ and S are one of the numbers ± 1 ; for odd electron number they are one of the numbers $\pm i$.

Since the states belonging to the μ -values named in (34) are nondegenerate, their number Z' is calculated according to (21), and afterwards the number $Z'' = \frac{1}{2}(2J+1-Z')$ of doubled¹⁷ terms.

The numerical distribution of the simple [nondegenerate] states over the reflection factors $S = \pm 1$ or $\pm i$ is given further through the following considerations:

For odd electron number N the number of simple terms is even. There are just as many terms with $S = -i$ as terms with $S = i$. Thus in all cases $p = 3$, $\mu = 3/2$ [correction per erratum] that occur¹⁸

$$Z_{3/2,1} = Z_{3/2,-1} = 1/2 Z' \quad (43)$$

For even electron number N the number Z' [correction per erratum] of simple states is odd, since here the case $M = 0$ plays an exceptional role, in that for $M = 0$ in Eq. (42) only the positive sign appears, of course. For $M = 0$ the reflection factor, according to Eq. (41), has the value

$$S = (-1)^{\Sigma(1-j)} = \begin{cases} 1 & \text{if } \Sigma(1-j) \text{ is even} \\ -1 & \text{if } \Sigma(1-j) \text{ is odd.} \end{cases} \quad (44)$$

The remaining even number of terms is distributed again evenly over both signs.

The degenerate states must yet be treated. While the nondegenerate states are characterized by a rotation factor $D(z)$ and a reflection factor S , degeneracy is just necessitated in that it is not always possible to construct states for which $D(z)$ and S are both defined [recall that $D(z)$ describes the p -fold rotation symmetry, and S the vertical reflection plane(s)]. Since it is always possible in the cyclic classes, according to the results of Section 4, to form $2J+1$ states with defined $D(z)$, one seeks effectively to construct states which have a well defined $D(z)$, i.e. μ , but transforming linearly among themselves under reflection in the zx -plane (S is not defined). Such states are, with respect to their symmetry, states of atoms in the cyclic classes which coincide under introduction of the reflection plane from symmetry bases. Or, inversely formulated, the collapsing states thus formed in the perturbing potential with reflection planes are those suited to the degeneracy-preserving additional perturbation, which just remains in the reduction of symmetry through elimination of the reflection planes. It should be noted equally that states belonging to degenerate terms never contain both ψ_M and ψ_{-M} , since such states, as proved above, are nondegenerate [the equation-system (41) for the mixed amplitudes is quadratic; the two solutions are nontrivially distinct, and they will in general be nondegenerate].

It is easily shown that degeneracy higher than two-fold never occurs, that is, that two states always transform into one another under reflection. That is to say

$$u_1 = a_{M1} \psi_{M1} + a_{M2} \psi_{M2} + \dots \quad (45)$$

a state of the cyclic class C_p with the rotation factor $D(z)_1 = \exp(i\mu 2\pi/p)$, so the state

$$u_2 = a_{-M1} \psi_{-M1} + a_{-M2} \psi_{-M2} + \dots \quad (46)$$

certainly has the rotation factor $D(z)_2 = \exp(-i\mu 2\pi/p)$. Under reflection, according to (15), u_1 goes over into

$$u_1(x-yz) = (-1)^{\Sigma 1} \cdot \exp i\pi(M_1 - \Sigma j) \cdot a_{M1} \cdot \psi_{M1}(xyz) + \dots \quad (47)$$

and u_2 goes into

$$u_2(x-yz) = (-1)^{\Sigma 1} \cdot \exp i\pi(-M_1 - \Sigma j) \cdot a_{-M1} \cdot \psi_{-M1}(xyz) + \dots$$

With that the states u_1 and u_2 coincide energetically; according to Section 3

$$\begin{aligned} u_1(x-yz) &= A_1 u_1(xyz) + A_2 u_2(xyz) \\ u_2(x-yz) &= B_1 u_1(xyz) + B_2 u_2(xyz) \end{aligned} \quad (49)$$

that is

$$\begin{aligned} &(-1)^{\sum l} \cdot \exp i \pi (M - \sum j) a_{M1} \psi_{-M1} + \dots \\ &= A_1 (a_{M1} \psi_{M1} + \dots) + A_2 (a_{-M1} \psi_{-M1} + \dots) \\ &(-1)^{\sum l} \cdot \exp i \pi (-M_1 - \sum j) a_{-M1} \psi_{M1} + \dots \\ &= B_1 (a_{M1} \psi_{M1} + \dots) + B_2 (a_{-M1} \psi_{-M1} + \dots) \end{aligned} \quad (50)$$

Equating coefficients yields immediately

$$A_1 = B_2 = 0, \quad |A_2| = |B_1| = 1 \quad (51)$$

That is, up to a factor, u_1 goes over into u_2 and vice versa. Further, there follows for all i

$$\begin{aligned} A_2 a_{-M1} &= (-1)^{\sum (1-j)+M1} \cdot a_{M1} \\ B_1 a_{M1} &= (-1)^{\sum (1-j)-M1} \cdot a_{-M1}, \end{aligned} \quad (52)$$

that is

$$a_{-M1} = 1/A_2 (-1)^{\sum (1-j)+M1} \cdot a_{M1} = B_1 (-1)^{\sum (j-1)+M1} \cdot a_{M1}. \quad (53)$$

$$B_1 A_2 = (-1)^{2\sum j} = \begin{cases} +1 & \text{for even } N \\ -1 & \text{for odd } N. \end{cases} \quad (54)$$

With that, u_1 and u_2 coincide; therefore the a_{-M1} need only stand in a constant proportion B_1 to the a_{M1} , except for the factor of $(-1)^{\sum (j-1)+M1}$. Then consequently for each non-simple state designated by μ , a second can be formed with opposite rotational sense about the z -axis, therefore characterized by $-\mu$, so that both mutually transform into each other; a degeneracy no higher than twofold is required by the symmetry. Therefore states with opposite rotation sign about the p -order covering axis always coincide energetically. Since these states have opposed signs of μ , the doubled terms should be characterized by the symbol $\pm\mu$.

Their number is given by [see again note 15 re square brackets]

$$z_{\pm\mu} = z_{\mu} = z_{-\mu} = 1 + [(J+\mu)/p] + [(J-\mu)/p], \quad (55)$$

and in conclusion the total number of components into which the term of the free atom splits is given by [nondegenerate + doubly degenerate]

$$Z = Z' + Z'' = Z' + 1/2 (2J+1-Z') = J + (Z'+1)/2. \quad (56)$$

It should yet be mentioned also that the homogeneous electrical field parallel to the z -axis belongs to the cyclic classes with reflection planes, as the limit case $p=\infty$. That is, the perturbing potential here is

$$V(z) = e E \sum_k z_k;$$

therefore it is invariant against all rotations about the z -axis and against reflection in each plane through the z -axis. That is, according to (17) $\mu=M$; and therefore, according to (34), only states with $M=0$ are nondegenerate. The states are the ψ_M ; and ψ_{-M} coincides energetically with ψ_M . Further there follow, in the limit, the correct values $Z_1=0$ for odd electron number N , and $Z'_1=1$ [correction per erratum] for even N .

8. Dihedral Classes, Column 5 from Table 1

Here there enters, besides the p-fold covering axis, a two-fold covering axis standing perpendicular to it. The y-axis is placed in it. Therefore, from simple [nondegenerate] states, besides compliance with Eq. (17), compliance is required with the additional relation

$$u_i(-x, y, -z) = D(y) u_i(xyz) \quad (57)$$

that is, under overturning of the coordinate system about the y-axis the state should go into itself up to a factor $D(y)$. Now Eq. (12) shows that under the overturning each ψ_μ goes into $\psi_{-\mu}$. Therefore the proportions are exactly as under the reflection plane treated in the previous section. Therefore the entire discussion proceeds exactly as for the cyclic classes with reflection planes going through the covering axis, with the single difference that the simple states are characterized not by $D(z)$ and S but by $D(z)$ and $D(y)$ (respectively μ and ν)-- wherein, according to Eq. (12) and (15), $D(y)$ is to be calculated from S without further details¹⁹

$$D(y) = S (-1)^{\Sigma i} = S \cdot I = \exp(-i\pi \Sigma j) = \exp i\nu\pi, \quad (58)$$

and wherein the "crystal quantum number" ν assigned to the crystallographic second axis has the range

$$\begin{aligned} \nu &= 0, 1 \text{ for even electron number } N. \\ \nu &= \pm \frac{1}{2} \text{ for odd electron number.} \end{aligned} \quad (59)$$

9. Cyclic-Inverse Classes with Reflection Planes, Column 6 from Table 1

For 2nd or 6th order inversion axes, reflection planes enter. The discussion proceeds exactly as for C_{2v} and C_{6v} . Splitting and component number are controlled through the value of μ_1 , being traced back to μ by means of Eq. (28), and the reflection factor S . Therefore the simple [nondegenerate] states are only those with μ -values which according to Eq. (28) correspond to μ -values $\mu=0$ or $p/2$.

10. Dihedral Classes with Inversion Centers, Column 7 from Table I

As in Section 6, here as well the new symmetry center joining the symmetry elements of the dihedral classes causes only an additional distinction between even and odd states.

11. Tabular Summation

In conclusion, for easier application to experiments, the results of the considerations are arranged tabularly-- actually in Table 2 for even electron numbers, and in Table 3 for odd. The tables give the individual term families for each symmetry class-- first in the specification ${}^g\Gamma_n$, according to the group theoretical work of Bethe and Opechowski, wherein g means the degree of degeneracy of the terms in the crystal and n is a running number (column 2), then in the specification by crystal quantum number (column 3). Column 4 gives the number of components in each family, into which a term with the rotational momentum quantum number J of the free atom splits; column 5 gives the symmetry class for that transition in which any still-degenerate components likewise get split. The quantum numbers used for characterization of the doubled terms are suited to this symmetry class.

pages 107-121

[A block of pages, 107 through mid-page 121... comprising Tables 2 and 3, has been transported entire from its position within the original German text, to an Appendix. This Appendix includes a translation of the headings to the Tables, as well as photocopies of the tables themselves from the German. Following the Appendix is an Erratum which was subsequently published on page 357 of the same volume.]

mid-page 121

III. Selection Rules for Electric Dipole Radiation

The electric dipole moment of the transition assigned from one state i to another state k is defined through the matrix element of its components. For example, its x-component is given by the scalar product

$$P_{x,ik} = e(u_i, x u_k) = e \sum_{s_1, \dots, s_n} \dots \sum_{s_1, \dots, s_n} \int \dots \int u_i^* x dx_1 \dots dx_n, \quad (60)$$

in which x stands as an abbreviation for $\sum_m x_m$, integrated over the coordinates for all electrons and summed over each value of the spin coordinate. In many cases it is expedient to calculate elliptically polarized radiation from the outset, the matrix element of the operator $e(x-iy)$ for the positive sign (dipole rotating from the positive x-axis to the positive y-axis) and $e(x+iy)$ for the negative sign.

12. Cyclic Classes

With help from the transformation properties (16) and (20) there first results

$$\begin{aligned} (P_x \pm iP_y) &= e(u_i(r\theta\phi), r \sin\theta e^{\pm i\phi} u_k(r\theta\phi)) \quad (61) \\ &= \exp(-\Delta\mu 2\pi/p \pm 2\pi p) \cdot e(u_i(r'\theta'\phi'), r' \sin\theta' e^{\pm i\phi'} u_k(r'\theta'\phi')) \\ &= e^{-i(\Delta\mu \mp 1)2\pi/p} (P_x \pm P_y)_{ik} \end{aligned}$$

with

$$\begin{aligned} \Delta\mu &= \mu_1 - u_k \\ -(p-1) &\leq \Delta\mu \leq p-1. \end{aligned} \quad (62)$$

From this it follows immediately that the matrix element can differ from zero only if the exponential function has the value 1. Therefore the selection rule holds:

$$P_x \pm iP_y \neq 0 \quad \text{only if} \quad \Delta\mu = \pm 1 \pmod{p}. \quad (63)$$

The upper sign delivers dipole in the negative rotational sense, and the lower in the positive sense-- that is, for observation of elliptically polarized light in the z-direction. For the whorled crystals ($p \geq 3$), in connection with which the directions more inclined toward one another in the xy-plane are equivalent, the polarization is naturally circular; that is

$$|P_{x1k}|^2 = |P_{y1k}|^2. \quad (64)$$

Entirely analogous to (61), there results

Eq. (65)...

$$P_{z1k} = e^{-i\Delta\mu 2\pi/p} P_{z1k}, \quad (65)$$

That is, the selection rule

$$P_{z1k} \neq 0 \quad \text{only if} \quad \Delta\mu = 0 \pmod{p}, \quad (66)$$

which, because of the limitation of $\Delta\mu$ by (18) and (19) is equivalent to equivalent to

$$\Delta\mu = 0. \quad (67)$$

In the triclinic case ($p=1$), in which all states belong to $\mu=0$, rules (63) and (66) are identical; that is the x-, y-, and z-components of the dipole can simultaneously differ from zero (dipole with three degrees of freedom). The basis for this behavior of course lies in the fact that in a triclinic crystal each direction is a single-fold axis. For $p>1$, on the other hand, a dipole with one degree of freedom parallel to the z-axis and a dipole with two degrees of freedom in the xy-plane are strictly excluded.

In conclusion, it must be mentioned that the case of the anomalous Zeeman effect in a homogeneous magnetic field for $p=\infty$ is also contained in these selection rules:

$$\begin{aligned} \Delta M &= \pm 1 \text{ for } \sigma\text{-components} \\ \Delta M &= 0 \text{ for } \pi\text{-components} \end{aligned} \quad (68)$$

13. Cyclic-Inverse Classes, Column 2 Table 1

The transformation requirement Eq. (24) yields

$$\begin{aligned} (P_x + iP_y)_{1k} &= \exp(-i(\Delta\mu_1 \mp 1)2\pi/p) (P_x \pm iP_y)_{1k} \\ P_{z1k} &= -\exp(-i\Delta\mu_1 2\pi/p) P_{z1k}, \end{aligned} \quad (69)$$

from which the following selection rules immediately arise:

$$\begin{aligned} (P_x + iP_y) &\neq 0 \quad \text{only if} \quad \Delta\mu_1 = 1 + p/2 \pmod{p} \\ (P_x - iP_y) &\neq 0 \quad \text{only if} \quad \Delta\mu_1 = -(1 + p/2) \pmod{p} \\ P_{z1k} &\neq 0 \quad \text{only if} \quad \Delta\mu_1 = p/2 \pmod{p}. \end{aligned} \quad (70)$$

One writes in accordance with Eq. (28)

$$\Delta\mu_1 = \Delta\mu + (p/2) \Delta \sum_k l_k \pmod{p}. \quad (71)$$

and in accordance with Eq. (30)

$$(-1)^{\Delta\sum l} = I_1 \cdot I_k, \quad (72)$$

Thus selection rule (70) can also be written in the following form²⁰:

$$\begin{aligned} (P_x \pm i P_y)_{1k} \neq 0 \text{ only if either } I_1 = -I_k, \Delta\mu = \pm 1 \pmod{p} \quad (73) \\ \text{or } I_1 = I_k, \Delta\mu = (1 + p/2) \pmod{p} \\ P_{x1k} \neq 0 \text{ only if either } I_1 = I_k, \Delta\mu = p/2 \pmod{p} \\ \text{or } I_1 = -I_k, \Delta\mu = p/2 \pmod{p}. \end{aligned}$$

If the order of the principal axis is odd ($p=1,3$), then of course each time only the first of the two possibilities comes into question, since $\Delta\mu$ can not be half-integral.

14. Cyclic Classes with Inversion Centers, Column 3 from Table 1

The covering axis requires compliance with the rules (63) and (66). The inversion center appearing additionally leads further to the following well-known rule: According to (29)

$$\begin{aligned} P_{x1k} = e(u_1(xyz), x u_k(xyz)) = -I_1 \cdot I_k \cdot e(u_1(-x, -y, -z), -x u_k(-x, -y, -z)) \\ = I_1 I_k P_{x1k} \end{aligned} \quad (74)$$

and correspondingly

$$\begin{aligned} P_{y1k} &= -I_1 I_k P_{y1k} \\ P_{z1k} &= -I_1 I_k P_{z1k} \end{aligned} \quad (75)$$

With that, therefore, for a spectral line to appear at all, there must be

$$I_1 = -I_k. \quad (76)$$

That is, even states combine only with odd and vice versa; the value of the sum $\sum_k l_k$ must change by an odd number in the transition (Laporte's Rule).

15. Cyclic Classes with Vertical Mirror Planes, Column 4 from Table 1

Transitions between nondegenerate states are treated first. Besides the μ -rule (63) and (66) demanded by the p -fold axis, there enters additionally a selection rule demanded by the mirror plane, which simultaneously must be obeyed and which arises in the following way. If one applies (31) to the components of the dipole moments²¹, there results

$$\begin{aligned}
 P_{x'ik} &= S_i^* \cdot S_k \cdot P_{xik} = I_i I_k e^{-i\Delta\nu\tau} \cdot P_{xik} \\
 P_{y'ik} &= -S_i^* \cdot S_k \cdot P_{yik} = -I_i I_k e^{-i\Delta\nu\tau} \cdot P_{yik} \\
 P_{z'ik} &= S_i^* \cdot S_k \cdot P_{zik} = I_i I_k e^{-i\Delta\nu\tau} \cdot P_{zik},
 \end{aligned}
 \quad (77)$$

in which Eq. (58) and $\Delta\nu = \nu_i - \nu_k$ are used on the right²².

That is, it can be

$$\begin{aligned}
 P_{x'ik} &\neq 0 \quad \text{only if } S_i = S_k \\
 P_{y'ik} &\neq 0 \quad \text{only if } S_i = -S_k \\
 P_{z'ik} &\neq 0 \quad \text{only if } S_i = S_k.
 \end{aligned}
 \quad (78)$$

Therefore either X_{ik} and Z_{ik} are allowed, that is the components lying in the mirror plane; and Y_{ik} , the component perpendicular to the mirror plane, vanishes-- or vice versa. The first is the case of transitions between states with equal reflection factor S ; the last is the case of transitions between states with unequal S . Therefore a dipole rotating in the xy-plane does not occur ($P_{x'ik}P_{y'ik} \neq 0$).

Thus in the case of degeneracy the coincident states are so determined, that they are states of the cyclic classes; here the rule for $\Delta\mu$, (63) and (66), is simply applied separately to the coincident transitions. The polarization of the resulting lines therefore results from the superposition of several individual lines of known polarization.

In the individual lines, the simultaneous compliance with the rotational and the reflectional rules results in the following picture:

a) $p=2$, C_{2v} . If the electron number N is even, then all states are nondegenerate. The $\Delta\mu$ -rule (63) and (66) requires that z- and x-components never occur together; the reflection rule (78) requires that x- and y- never occur together, as well as y- and z-components. Each dipole is therefore oriented parallel to one of the three rhombic axes. On the other hand, if the electron number N is odd, then all states are double; and there is only the $\Delta\mu$ -rule to be applied. Since two states with $\mu=\pm\frac{1}{2}$ coincide, dipoles oriented parallel and perpendicular to the 2-fold axis coincide; that is, the resulting dipoles have in general a non-vanishing component for each of the three rhombic axes (dipole with three degrees of freedom). The intensity distribution for the three components depends entirely upon the special form and magnitude of the perturbational field in the individual case at hand.

b) Whorled classes, $p \geq 3$. Since all nondegenerate states belong to $\mu=0$ or $\mu=p/2 > 1$, only transitions with $\Delta\mu=0$ occur between them; that is, in all cases the dipole lies parallel to the covering axis. Through the condition (78) the number of allowed transitions is further diminished, to those for which the reflection factor does not change. Transitions to degenerate terms, for which all the rules (63) and (66) hold, lead to dipoles with two degrees of freedom in the plane perpendicular to the axis, because of the coincidence of differently polarized transitions-- or even to dipoles with three spatial degrees of freedom.

16. Dihedral Classes, Column 5 from Table 1

The degeneracy is the same as for the cyclic classes with vertical reflection planes, except that the rotation factor $D(y)$ enters in the place of the reflection factor S . With use of Eq. (57) and (58), there results for the components of the dipole moment

$$\begin{aligned} P_{xik} &= -D^*(y)_i D(y)_k & P_{xik} &= -e^{-i\Delta\nu/\hbar} \cdot P_{xik} \\ P_{yik} &= D^*(y)_i D(y)_k & P_{yik} &= -e^{-i\Delta\nu/\hbar} \cdot P_{yik} \\ P_{zik} &= -D^*(y)_i D(y)_k & P_{zik} &= -e^{-i\Delta\nu/\hbar} \cdot P_{zik} \end{aligned} \quad (79)$$

That is, there can be

$$P_{xik} \neq 0 \text{ and } P_{zik} \neq 0 \text{ only if } D(y)_i = -D(y)_k, \text{ that is } \Delta\nu = \pm 1. \quad (80)$$

$$P_{yik} \neq 0 \text{ only if } D(y)_i = D(y)_k, \text{ that is } \Delta\nu = 0.$$

Since, on the basis of the $\Delta\mu$ -rule, the dipole corresponding to a transition between nondegenerate states always lies parallel to the z-axis, the $D(y)$ or ν must therefore change in the transition. For $p=2$, (D_2 symmetry), each dipole lies, as for C_{2v} , parallel to one of the rhombic axes.

17. Cyclic-Inverse Classes with Reflection Planes, Column 6 of Table 1

In addition to rule (73), required by the rotation-inversion axis, there enters the reflection plane rule (78). Also here transitions between nondegenerate states always correspond to dipoles parallel to the z-axis. Therefore the part of rule (73) and (78) valid for the z-component must be fulfilled.

18. Dihedral Classes with Symmetry Center, Column 7 from Table 1

The symmetry center requires, in addition to (63), (65), and (80), compliance to the Laporte rule (76).

IV. Domain of Application of the Theory

The theory developed above permits application without further ado to both the following limiting cases of coupling, already distinguished by Bethe [and Opechowski?]:

1. The crystal field splitting is small compared to the multiplet splitting. That is, the perturbation by the crystal is small compared to the spin-orbit interaction. Then the quantum number J is also, to a good approximation, still defined in the crystal. And the splitting of the term is provided by a perturbational calculation, in which the spin-orbit interaction in the final states already is accounted for in zeroth approximation. This case is implemented above; it is realized, for example, in the salts of the rare earths.

2. The crystal field splitting is, conversely, large compared with the multiplet splitting; that is the perturbation of the term by the crystal field is large compared with the spin-orbit interaction. Actually it should be even be regarded as so weak, compared to the Coulomb interaction of the individual electrons, that the quantum numbers S and S are also still defined in the crystal, to a good approximation. This case is realized, for example, for the salts of the iron row of the periodic system. Therefore we completely neglect, for the time being, the spin-orbit interaction-- as well as, for instance, any very weak magnetic portion of the crystal field, such as is certainly present. Then, in zeroth approximation, the states in the crystal are simple products of one spatial function to be determined from the orbital momentum L and a spin function

$$u_i(x_1 \dots x_N, s_1 \dots s_N) = u_{L_i}(x_1 \dots x_N) \cdot u_{S_i}(s_1 \dots s_N) \quad (81)$$

page 126

And, to be stated before turning on the crystal field, there are $(2L+1)(2S+1)$ such cases, states together belonging to the pretended values of L and S . The question, in accordance with the symmetry properties of these functions and according to the degree of splitting under turning-on of the crystal field, is answered in the following way: Since the crystal field, for the time being imagined purely electric, operates noticeably only on the orbital motion of the electrons, but not on the spin, the theory developed above is first applied to the orbit alone, in that one identifies L with J . If one allows the spin orbit interaction and the magnetic portion of the crystal field to take effect only in higher approximation, then the spin states split up weakly and one has to apply the theory for a second time, in that now identifies S with J ; that is each of the $(2L+1)(2S+1)$ states is characterized by symmetry factors

$$D_L(z) = \exp i\mu_L 2\pi, D_L(y) = \exp i\nu_L \pi, S_L, I_L \quad (82)$$

of the orbit, and further by symmetry factors
Eq. (83)...

$$D_S(z) = \exp i\mu_S 2\pi, D_S(y) = \exp i\nu_S \pi, S_S, I_S \quad (83)$$

of the spins. If one goes back again to the Pauli-state of a spin electron, then one obtains the symmetry factors of the orbit (82) when one sets

$$j_k = l_k \text{ and } m_k = m_{l_k}; \quad (84)$$

in the general formula written above, and the symmetry factors of the spin (83) when one sets

$$j_k = \frac{1}{2}, l_k = 0, m_k = m_{s_k}. \quad (85)$$

The matrix elements $P_{q,ik}(q=x,y,z)$ for electric dipole radiation come to light, likewise because of (81), as products of an orbital and a spin factor

$$P_{q,ik} = (u_{L_i}, u_{S_i}, q u_{L_k}, u_{S_k}) = \sum_{s_1} \dots \sum_{s_N} u_{s_1}^* u_{s_k} \int \dots \int u_{L_i}^* q u_{L_k} dx_1 \dots dx_N. \quad (86)$$

In order that there can be $P_{q,ik} \neq 0$, neither of the two factors may vanish. First for the orbital portion, this means that the selection rules derived in Section III hold for its symmetry factors (82). For the spin factor it means, because of the orthogonality of the $\epsilon(s)$ and $\eta(s)$, that u_{s_i} and u_{s_k} must be the same functions up to the value of the constant factors. That is, it holds for the moment with the sharpness with which S^{23} , that intercombination is forbidden

$$\Delta S = 0; \quad (87)$$

and further the rules hold

$$\Delta \mu_S = 0, \Delta \nu_S = 0, S_i = S_k. \quad (88)$$

In the limit $p \rightarrow \infty$ the first of these rules goes over into the known rule

$$\Delta M_S = \Delta \sum_{k=1}^N m_{s_k} = 0$$

of the Paschen-Back effect, in which spin-orbit decoupling is caused by a homogeneous field.

Göttingen, II. Institute of Physics
(Arrived at the editor's office Mar. 8, 1947)

FOOTNOTES, REFERENCES

1. H. Bethe, Ann. Physik (5) 3, 133 (1929).
2. W. Opechowski, Physica 7, 552 (1940).
3. H. Bethe, Z. Physik 60, 218 (1930).
4. F. Hund, Z. Physik 43, 788 (1927); 99, 119 (1936); Hdb. d. Phys. 2. Aufl. Bd. XXIV₁.
5. The 5 cubic classes were treated in a later work.
6. The selection rules for magnetic dipole radiation and electric quadrupole radiation are found in two later works.
7. This part of the work therefore completes earlier investigations, in which for unknown microfield, solely from the symmetry of the total crystal, the cases compatible with this of different polarization of the spectral lines are deduced. K. H. Hellwege, Z. Physik 119, 588 (1943).
8. See for example B. W. Pauli in Hdb. d. Physik 2. Aufl. Bd. XXIV₁.
9. See for example Hdb. Physik, 2. Aufl., Bd. XXIV₁, Art. Pauli u. Bethe.
10. Definition as in the Handbook article by Bethe.
11. One finds this relationship by placing a magnetic field simply parallel to z and simply parallel to z' ; and both the states occurring for a stationary spin-electron, that is parallel and antiparallel to the field, at the same time described in the other coordinate system.
12. For the formal basis, see for example van der Waerden, Group Theoretical Methods in Quantum Mechanics, S. 92.
13. For M this holds strictly for interaction of arbitrary strength between the electrons; for the l_k and j_k [it holds] only for vanishing interaction.
14. As shown incidentally already in Equation (5).
15. The square brackets mean: Fractional numbers are to be rounded off downward.
16. Instead of the one mirror plane used here there are actually always p present, passing through the z -axis. It suffices, however, to treat only both demands (16) and (31). Quite generally in all classes, it suffices to treat only those floor operations independent of one another, from which the others are assembled.
17. Here already, the fact is anticipated that all non single-fold terms are doubly degenerate; see S. 105.
18. The first index gives the value of μ , [and] the second the value of S .
19. Naturally only $D(y)$, and ν respectively, are defined in the crystal. S and I here are pure operands, which have no intrinsic physical meaning, since the symmetry contains neither reflection planes nor inversion center.

20. μ and I function as pure operands.

21. At this juncture it is advantageous to treat the x - and y -components separately, and not in the combination of Eq. (65).

22. Here I and ν have only the character of operands.

23. There is surely no fear of confusion between the spin quantum number S and the reflection quantum number S_r .

APPENDIX: Tables 2 and 3

[Note that a block of pages, 107 through mid-page 121, has been transported entire from its position within the original German text, to form this appendix.

A translation of the headings follows immediately, and then photocopies of the tables from the German.

Errata are cited on pages 110 and 113 of these tables; see Errata following these tables.]

pages 107-121

[For Table 2, beginning on p. 107, and for its continuation pages ("Forsetzung") through p. 114-- and similarly for Table 3, beginning on p. 115, and its continuations through p. 121-- the headings are:]

Table 2
Term Splitting for Even Electron Number

1	2	3	4	5
Symmetry	According to	Term Families	Number of Terms	Complete
Classes	Group Theory	According to	in Each Family	Splitting
		Crystal	for Given J	In
		Quantum Number		
.....See tables following, from original German.....				

[See ref. 15 for the meaning of square brackets within these tables.]

Tabelle 2
Term aufspaltung bei gerader Elektronenzahl

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
C_1	$^1\Gamma_1$	$\mu = 0$	$2J + 1$	C_1
C_i	$^1\Gamma_1$	$\mu_I = 0$	$\frac{1}{2} \left(1 + (-1)^k \right) (2J + 1)$	C_i
	$^1\Gamma_2$	$\frac{1}{2}$	$\frac{1}{2} \left(1 - (-1)^k \right) (2J + 1)$	
C_2	$^1\Gamma_1$	$\mu = 0$	$1 + 2 \left[\frac{J}{2} \right]$	C_2
	$^1\Gamma_2$	1	$1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right]$	
C_s	$^1\Gamma_1$	$\mu_I = 0$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + 2 \left[\frac{J}{2} \right] \right)$	C_s
	$^1\Gamma_2$	1	$\frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	
C_{2h}	$^1\Gamma_1$	$I = 1, \mu = 0$	$\frac{1}{2} (1 + I) \left(1 + 2 \left[\frac{J}{2} \right] \right)$	C_{2h}
	$^1\Gamma_2$	1, 1	$\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	
	$^1\Gamma_3$	-1, 0	$\frac{1}{2} (1 - I) \left(1 + 2 \left[\frac{J}{2} \right] \right)$	
	$^1\Gamma_4$	-1, 1	$\frac{1}{2} (1 - I) \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	
C_{2v}	$^1\Gamma_1$	$S = 1, \mu = 0$	$\left[\frac{J}{2} \right] + \frac{1}{2} \left(1 + (-1)^k \right) \Sigma^{(I_k - J_k)}$	C_{2v}
	$^1\Gamma_2$	-1, 0	$\left[\frac{J}{2} \right] + \frac{1}{2} \left(1 - (-1)^k \right) \Sigma^{(I_k - J_k)}$	
	$^1\Gamma_3$	1, 1	$\frac{1}{2} \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	
	$^1\Gamma_4$	-1, 1	$\frac{1}{2} \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	
D_2	$^1\Gamma_1$	$\nu = 0, \mu = 0$	$\left[\frac{J}{2} \right] + \frac{1}{2} \left(1 + (-1)^k \right) \Sigma^{J_k}$	D_2
	$^1\Gamma_2$	1, 0	$\left[\frac{J}{2} \right] + \frac{1}{2} \left(1 - (-1)^k \right) \Sigma^{J_k}$	
	$^1\Gamma_3$	0, 1	$\frac{1}{2} \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	
	$^1\Gamma_4$	1, 1	$\frac{1}{2} \left(1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right)$	

Tabelle 2 (Fortsetzung)

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
D_{2h}	1F_1	$I = 1, \nu = 0, \mu = 0$	$\frac{1}{2}(1+I) \left\{ \left[\frac{J}{2} \right] + \frac{1}{2} (1+(-1)^k \Sigma^{ik}) \right\}$	D_{2h}
	1F_2	1, 1, 0	$\frac{1}{2}(1+I) \left\{ \left[\frac{J}{2} \right] + \frac{1}{2} (1-(-1)^k \Sigma^{ik}) \right\}$	
	1F_3	1, 0, 1	$\frac{1}{4}(1+I) \left\{ 1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right\}$	
	1F_4	1, 1, 1	$\frac{1}{4}(1+I) \left\{ 1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right\}$	
	1F_5	-1, 0, 0	$\frac{1}{2}(1-I) \left\{ \left[\frac{J}{2} \right] + \frac{1}{2} (1+(-1)^k \Sigma^{ik}) \right\}$	
	1F_6	-1, 1, 0	$\frac{1}{2}(1-I) \left\{ \left[\frac{J}{2} \right] + \frac{1}{2} (1-(-1)^k \Sigma^{ik}) \right\}$	
	1F_7	-1, 0, 1	$\frac{1}{4}(1-I) \left\{ 1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right\}$	
	1F_8	-1, 1, 1	$\frac{1}{4}(1-I) \left\{ 1 + \left[\frac{J+1}{2} \right] + \left[\frac{J-1}{2} \right] \right\}$	
C_3	1F_1	$\mu = 0$	$1 + 2 \left[\frac{J}{3} \right]$	C_3
	1F_2	1	$1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right]$	
	1F_3	-1	$1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right]$	
C_{3i}	1F_1	$\mu_i = 0$	$\frac{1}{2} (1+(-1)^k \Sigma^{ik}) \left(1 + 2 \left[\frac{J}{3} \right] \right)$	C_{3i}
	1F_2	1	$\frac{1}{2} (1+(-1)^k \Sigma^{ik}) \left(1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right] \right)$	
	1F_3	-1	$\frac{1}{2} (1+(-1)^k \Sigma^{ik}) \left(1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right] \right)$	
	1F_4	$\frac{3}{2}$	$\frac{1}{2} (1-(-1)^k \Sigma^{ik}) \left(1 + 2 \left[\frac{J}{3} \right] \right)$	
	1F_5	$\frac{1}{2}$	$\frac{1}{2} (1-(-1)^k \Sigma^{ik}) \left(1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right] \right)$	
	1F_6	$-\frac{1}{2}$	$\frac{1}{2} (1-(-1)^k \Sigma^{ik}) \left(1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right] \right)$	
C_{3v}	1F_1	$S = 1, \mu = 0$	$\left[\frac{J}{3} \right] + \frac{1}{2} (1+(-1)^k \Sigma^{(ik-jk)})$	C_{3v}
	1F_2	-1, 0	$\left[\frac{J}{3} \right] + \frac{1}{2} (1-(-1)^k \Sigma^{(ik-jk)})$	
	1F_3	± 1	$1 + \left[\frac{J+1}{3} \right] + \left[\frac{J-1}{3} \right]$	

Tabelle 2 (Fortsetzung)

1 Sym- me- tric- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
D_3	${}^1\Gamma_1$	$\nu = 0, \mu = 1$	$\left[\frac{J}{3}\right] + \frac{1}{2} \left(1 + (-1)^{\sum i_k}\right)$	C_3
	${}^1\Gamma_2$	1, 0	$\left[\frac{J}{3}\right] + \frac{1}{2} \left(1 - (-1)^{\sum i_k}\right)$	
	${}^2\Gamma_3$	± 1	$1 + \left[\frac{J+1}{3}\right] + \left[\frac{J-1}{3}\right]$	
D_{3d}	${}^1\Gamma_1$	$I = 1, \nu = 0, \mu = 0$	$\frac{1}{2} (1 + I) \left\{ \left[\frac{J}{3}\right] + \frac{1}{2} \left(1 + (-1)^{\sum i_k}\right) \right\}$	C_3
	${}^1\Gamma_2$	1, 1, 0	$\frac{1}{2} (1 + I) \left\{ \left[\frac{J}{3}\right] + \frac{1}{2} \left(1 - (-1)^{\sum i_k}\right) \right\}$	
	${}^2\Gamma_3$	1, ± 1	$\frac{1}{2} (1 + I) \left\{ 1 + \left[\frac{J+1}{3}\right] + \left[\frac{J-1}{3}\right] \right\}$	
	${}^1\Gamma_4$	-1, 0, 0	$\frac{1}{2} (1 - I) \left\{ \left[\frac{J}{3}\right] + \frac{1}{2} \left(1 + (-1)^{\sum i_k}\right) \right\}$	
	${}^1\Gamma_5$	-1, 1, 0	$\frac{1}{2} (1 - I) \left\{ \left[\frac{J}{3}\right] + \frac{1}{2} \left(1 - (-1)^{\sum i_k}\right) \right\}$	
	${}^2\Gamma_6$	-1, ± 1	$\frac{1}{2} (1 - I) \left\{ 1 + \left[\frac{J+1}{3}\right] + \left[\frac{J-1}{3}\right] \right\}$	
C_4	${}^1\Gamma_1$	$\mu = 0$	$1 + 2 \left[\frac{J}{4}\right]$	C_4
	${}^1\Gamma_3$	1	$1 + \left[\frac{J+1}{4}\right] + \left[\frac{J-1}{4}\right]$	
	${}^1\Gamma_4$	-1	$1 + \left[\frac{J+1}{4}\right] + \left[\frac{J-1}{4}\right]$	
	${}^1\Gamma_2$	2	$1 + \left[\frac{J+2}{4}\right] + \left[\frac{J-2}{4}\right]$	
S_4	${}^1\Gamma_1$	$\mu_1 = 0$	$\frac{1}{2} \left(1 + (-1)^{\sum i_k}\right) \left(1 + 2 \left[\frac{J}{4}\right]\right)$ $+ \frac{1}{2} \left(1 - (-1)^{\sum i_k}\right) \left(1 + \left[\frac{J+2}{4}\right] + \left[\frac{J-2}{4}\right]\right)$	S_4
	${}^1\Gamma_3$	1	$1 + \left[\frac{J+1}{4}\right] + \left[\frac{J-1}{4}\right]$	
	${}^1\Gamma_4$	-1	$1 + \left[\frac{J+1}{4}\right] + \left[\frac{J-1}{4}\right]$	
	${}^1\Gamma_2$	2	$\frac{1}{2} \left(1 + (-1)^{\sum i_k}\right) \left(1 + \left[\frac{J+2}{4}\right] + \left[\frac{J-2}{4}\right]\right)$ $+ \frac{1}{2} \left(1 - (-1)^{\sum i_k}\right) \left(1 + 2 \left[\frac{J}{4}\right]\right)$	

Tabelle 2 (Fortsetzung)

1 Sym- me- trie- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
C_{4h}	1F_1	$I = 1, \quad \mu = 0$	$\frac{1}{2}(1+I)\left(1+2\left[\frac{J}{4}\right]\right)$	C_{4h}
	1F_3	1, 1	$\frac{1}{2}(1+I)\left(1+\left[\frac{J+1}{4}\right]+\left[\frac{J-1}{4}\right]\right)$	
	1F_4	1, -1	$\frac{1}{2}(1+I)\left(1+\left[\frac{J+1}{4}\right]+\left[\frac{J-1}{4}\right]\right)$	
	1F_2	1, 2	$\frac{1}{2}(1+I)\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$	
	1F_5	-1, 0	$\frac{1}{2}(1-I)\left(1+2\left[\frac{J}{4}\right]\right)$	
	1F_7	-1, 1	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+1}{4}\right]+\left[\frac{J-1}{4}\right]\right)$	
	1F_8	-1, -1	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+1}{4}\right]+\left[\frac{J-1}{4}\right]\right)$	
	1F_6	-1, 2	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$	
C_{4v}	1F_1	$S = 1, \mu = 0$	$\left[\frac{J}{4}\right] + \frac{1}{2}\left(1+(-1)^k \sum (l_k - j_k)\right)$	C_4
	1F_2	-1, 0	$\left[\frac{J}{4}\right] + \frac{1}{2}\left(1-(-1)^k \sum (l_k - j_k)\right)$	
	1F_3	1, 2	$\frac{1}{2}\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$	
	1F_4	-1, 2	$\frac{1}{2}\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$	
	2F_5	± 1	$1+\left[\frac{J+1}{4}\right]+\left[\frac{J-1}{4}\right]$	
D_4	1F_1	$\nu = 0, \mu = 0$	$\left[\frac{J}{4}\right] + \frac{1}{2}\left(1+(-1)^k \sum j_k\right)$	C_4
	1F_2	1, 0	$\left[\frac{J}{4}\right] + \frac{1}{2}\left(1-(-1)^k \sum j_k\right)$	
	1F_3	0, 2	$\frac{1}{2}\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$	
	1F_4	1, 2	$\frac{1}{2}\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$	
	2F_5	± 1	$1+\left[\frac{J+1}{4}\right]+\left[\frac{J-1}{4}\right]$	
See Erratum	D_{2d}	$S = 1, \mu_I = 0$	$\frac{1}{4}\left(1+(-1)^k \sum l_k\right)\left(1+2\left[\frac{J}{4}\right]\right)$ $+ \frac{1}{4}\left(1-(-1)^k \sum l_k\right)\left(1+\left[\frac{J+2}{4}\right]+\left[\frac{J-2}{4}\right]\right)$ $+ \frac{1}{2}\left(1+(-1)^k \sum (l_k - j_k)\right)$	

Tabelle 2 (Fortsetzung)

1 Sym- me- tric- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^1\Gamma_2$	$S = -1, \mu_I = 0$	$\frac{1}{4} \left(1 + (-1)^{\sum i_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum i_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $+ \frac{1}{2} \left(1 - (-1)^{\sum (i_k - j_k)} \right)$	
	${}^1\Gamma_3$	1, 2	$\frac{1}{4} \left(1 + (-1)^{\sum i_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum i_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$	
	${}^1\Gamma_4$	-1, 2	$\frac{1}{4} \left(1 + (-1)^{\sum i_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum i_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$	
	${}^2\Gamma_5$	± 1	$1 + \left[\frac{J+1}{4} \right] + \left[\frac{J-1}{4} \right]$	S_4
D_{4h}	${}^1\Gamma_1$	$I = 1, v = 0, \mu = 0$	$\frac{1}{2} (1 + I) \left\{ \left[\frac{J}{4} \right] + \frac{1}{2} \left(1 + (-1)^{\sum j_k} \right) \right\}$	
	${}^1\Gamma_2$	1, 1, 0	$\frac{1}{2} (1 + I) \left\{ \left[\frac{J}{4} \right] + \frac{1}{2} \left(1 - (-1)^{\sum j_k} \right) \right\}$	
	${}^1\Gamma_3$	1, 0, 2	$\frac{1}{4} (1 + I) \left\{ 1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right\}$	
	${}^1\Gamma_4$	1, 1, 2	$\frac{1}{4} (1 + I) \left\{ 1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right\}$	
	${}^2\Gamma_5$	1, ± 1	$\frac{1}{2} (1 + I) \left\{ 1 + \left[\frac{J+1}{4} \right] + \left[\frac{J-1}{4} \right] \right\}$	
	${}^1\Gamma_6$	-1, 0, 0	$\frac{1}{2} (1 - I) \left\{ \left[\frac{J}{4} \right] + \frac{1}{2} \left(1 + (-1)^{\sum j_k} \right) \right\}$	
	${}^1\Gamma_7$	-1, 1, 0	$\frac{1}{2} (1 - I) \left\{ \left[\frac{J}{4} \right] + \frac{1}{2} \left(1 - (-1)^{\sum j_k} \right) \right\}$	
	${}^1\Gamma_8$	-1, 0, 2	$\frac{1}{4} (1 - I) \left\{ 1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right\}$	
	${}^1\Gamma_9$	-1, 1, 2	$\frac{1}{4} (1 - I) \left\{ 1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right\}$	
	${}^2\Gamma_{10}$	-1, ± 1	$\frac{1}{2} (1 - I) \left\{ 1 + \left[\frac{J+1}{4} \right] + \left[\frac{J-1}{4} \right] \right\}$	C_{4h}
U_4	${}^1\Gamma_1$	$\mu = 0$	$1 + 2 \left[\frac{J}{6} \right]$	
	${}^1\Gamma_3$	1	$1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right]$	
	${}^1\Gamma_4$	-1	$1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right]$	

Tabelle 2 (Fortsetzung)

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in	
	${}^1\Gamma_5$ ${}^1\Gamma_6$ ${}^1\Gamma_2$	$\mu = 2$ -2 3	$1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right]$ $1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right]$ $1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right]$	C_6	
C_{3h}	${}^1\Gamma_1$ ${}^1\Gamma_3$ ${}^1\Gamma_4$ ${}^1\Gamma_5$ ${}^1\Gamma_6$ ${}^1\Gamma_2$	$\mu_1 = 0$ 1 -1 2 -2 3	$\frac{1}{2} (1 + (-1)^{\sum i_k}) \left(1 + 2 \left[\frac{J}{6} \right] \right)$ $+ \frac{1}{2} (1 - (-1)^{\sum i_k}) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $\frac{1}{2} (1 + (-1)^{\sum i_k}) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $+ \frac{1}{2} (1 - (-1)^{\sum i_k}) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $\frac{1}{2} (1 + (-1)^{\sum i_k}) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $+ \frac{1}{2} (1 - (-1)^{\sum i_k}) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $\frac{1}{2} (1 + (-1)^{\sum i_k}) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $+ \frac{1}{2} (1 - (-1)^{\sum i_k}) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $\frac{1}{2} (1 + (-1)^{\sum i_k}) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $+ \frac{1}{2} (1 - (-1)^{\sum i_k}) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $\frac{1}{2} (1 + (-1)^{\sum i_k}) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{2} (1 - (-1)^{\sum i_k}) \left(1 + 2 \left[\frac{J}{6} \right] \right)$	C_{3h}	
C_{6h}	${}^1\Gamma_1$ ${}^1\Gamma_3$ ${}^1\Gamma_4$ ${}^1\Gamma_5$ ${}^1\Gamma_6$ ${}^1\Gamma_2$	$I = 1,$ $1,$ $1,$ $1,$ $1,$ $1,$	$\mu = 0$ 1 -1 2 -2 3	$\frac{1}{2} (1 + I) \left(1 + 2 \left[\frac{J}{6} \right] \right),$ $\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$	

Tabelle 2 (Fortsetzung)

1 Sym- me- tric- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^1\Gamma_7$	$I = -1, \quad \mu = 0$	$\frac{1}{2}(1-I)\left(1+2\left[\frac{J}{6}\right]\right)$	C_{6h}
	${}^1\Gamma_9$	$-1, \quad 1$	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+1}{6}\right]+\left[\frac{J-1}{6}\right]\right)$	
	${}^1\Gamma_{10}$	$-1, \quad -1$	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+1}{6}\right]+\left[\frac{J-1}{6}\right]\right)$	
	${}^1\Gamma_{11}$	$-1, \quad 2$	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+2}{6}\right]+\left[\frac{J-2}{6}\right]\right)$	
	${}^1\Gamma_{12}$	$-1, \quad -2$	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+2}{6}\right]+\left[\frac{J-2}{6}\right]\right)$	
	${}^1\Gamma_8$	$-1, \quad 3$	$\frac{1}{2}(1-I)\left(1+\left[\frac{J+3}{6}\right]+\left[\frac{J-3}{6}\right]\right)$	
C_{6v}	${}^1\Gamma_1$	$S = 1, \mu = 0$	$\left[\frac{J}{6}\right] + \frac{1}{2}\left(1+(-1)^k \sum (i_k - j_k)\right)$	C_6
	${}^1\Gamma_2$	$-1, \quad 0$	$\left[\frac{J}{6}\right] + \frac{1}{2}\left(1-(-1)^k \sum (i_k - j_k)\right)$	
	${}^1\Gamma_3$	$1, \quad 3$	$\frac{1}{2}\left(1+\left[\frac{J+3}{6}\right]+\left[\frac{J-3}{6}\right]\right)$	
	${}^1\Gamma_4$	$-1, \quad 3$	$\frac{1}{2}\left(1+\left[\frac{J+3}{6}\right]+\left[\frac{J-3}{6}\right]\right)$	
	${}^2\Gamma_6$	± 1	$1+\left[\frac{J+1}{6}\right]+\left[\frac{J-1}{6}\right]$	
	${}^2\Gamma_5$	± 2	$1+\left[\frac{J+2}{6}\right]+\left[\frac{J-2}{6}\right]$	
D_6	${}^1\Gamma_1$	$v = 0, \mu = 0$	$\left[\frac{J}{6}\right] + \frac{1}{2}\left(1+(-1)^k \sum j_k\right)$	C_6
	${}^1\Gamma_2$	$1, \quad 0$	$\left[\frac{J}{6}\right] + \frac{1}{2}\left(1-(-1)^k \sum j_k\right)$	
	${}^1\Gamma_3$	$0, \quad 3$	$\frac{1}{2}\left(1+\left[\frac{J+3}{6}\right]+\left[\frac{J-3}{6}\right]\right)$	
	${}^1\Gamma_4$	$1, \quad 3$	$\frac{1}{2}\left(1+\left[\frac{J+3}{6}\right]+\left[\frac{J-3}{6}\right]\right)$	
	${}^2\Gamma_6$	± 1	$1+\left[\frac{J+1}{6}\right]+\left[\frac{J-1}{6}\right]$	
	${}^2\Gamma_5$	± 2	$1+\left[\frac{J+2}{6}\right]+\left[\frac{J-2}{6}\right]$	
See Erratum	D_{3h}	$S = 1, \mu_I = 0$	$\frac{1}{4}\left(1+(-1)^k \sum i_k\right)\left(1+2\left[\frac{J}{6}\right]\right)$ $+\frac{1}{4}\left(1-(-1)^k \sum i_k\right)\left(1+\left[\frac{J+3}{6}\right]+\left[\frac{J-3}{6}\right]\right)$ $+\frac{1}{2}\left(1+(-1)^k \sum (i_k - j_k)\right)$	

Tabelle 2 (Fortsetzung)

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^1\Gamma_2$	$S = -1, \mu_I = 0$	$\frac{1}{4} \left(1 + (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{2} \left(1 - (-1)^{\frac{\Sigma (I_k - j_k)}{k}} \right)$	
	${}^1\Gamma_3$	1, 3	$\frac{1}{4} \left(1 + (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$	
	${}^1\Gamma_4$	-1, 3	$\frac{1}{4} \left(1 + (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$	
	${}^2\Gamma_6$	± 1	$\frac{1}{2} \left(1 + (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $+ \frac{1}{2} \left(1 - (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$	
	${}^2\Gamma_8$	± 2	$\frac{1}{2} \left(1 + (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $+ \frac{1}{2} \left(1 - (-1)^{\frac{\Sigma I_k}{k}} \right) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$	C_{3A}
D_{6A}	${}^1\Gamma_1$	$I = 1, \nu = 0, \mu = 0$	$\frac{1}{2} (1 + I) \left\{ \left[\frac{J}{6} \right] + \frac{1}{2} \left(1 + (-1)^{\frac{\Sigma j_k}{k}} \right) \right\}$	
	${}^1\Gamma_2$	1, 1, 0	$\frac{1}{2} (1 + I) \left\{ \left[\frac{J}{6} \right] + \frac{1}{2} \left(1 - (-1)^{\frac{\Sigma j_k}{k}} \right) \right\}$	
	${}^1\Gamma_3$	1, 0, 3	$\frac{1}{4} (1 + I) \left\{ 1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right\}$	
	${}^1\Gamma_4$	1, 1, 3	$\frac{1}{4} (1 + I) \left\{ 1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right\}$	
	${}^2\Gamma_6$	1, ± 1	$\frac{1}{2} (1 + I) \left\{ 1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right\}$	
	${}^2\Gamma_8$	1, ± 2	$\frac{1}{2} (1 + I) \left\{ 1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right\}$	
	${}^1\Gamma_7$	-1, 0, 0	$\frac{1}{2} (1 - I) \left\{ \left[\frac{J}{6} \right] + \frac{1}{2} \left(1 + (-1)^{\frac{\Sigma j_k}{k}} \right) \right\}$	
	${}^1\Gamma_8$	-1, 1, 0	$\frac{1}{2} (1 - I) \left\{ \left[\frac{J}{6} \right] + \frac{1}{2} \left(1 - (-1)^{\frac{\Sigma j_k}{k}} \right) \right\}$	
	${}^1\Gamma_9$	-1, 0, 3	$\frac{1}{4} (1 - I) \left\{ 1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right\}$	
	${}^1\Gamma_{10}$	-1, 1, 3	$\frac{1}{4} (1 - I) \left\{ 1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right\}$	
	${}^2\Gamma_{12}$	-1, ± 1	$\frac{1}{2} (1 - I) \left\{ 1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right\}$	
	${}^2\Gamma_{11}$	-1, ± 2	$\frac{1}{2} (1 - I) \left\{ 1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right\}$	C_{6A}

Tabelle 3
Term aufspaltung bei ungerader Elektronenzahl

1 Sym- me- trie- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
C_1	${}^1\Gamma_2$	$\mu = \frac{1}{2}$	$2J+1$	C_1
C_i	${}^1\Gamma_3$	$\mu_I = \frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^{\sum k} \right) (2J+1)$	C_i
	${}^1\Gamma_4$	0	$\frac{1}{2} \left(1 - (-1)^{\sum k} \right) (2J+1)$	
C_2	${}^1\Gamma_3$	$\mu = \frac{1}{2}$	$J + \frac{1}{2}$	C_2
	${}^1\Gamma_4$	$-\frac{1}{2}$	$J + \frac{1}{2}$	
C_s	${}^1\Gamma_3$	$\mu_I = \frac{1}{2}$	$J + \frac{1}{2}$	C_s
	${}^1\Gamma_4$	$-\frac{1}{2}$	$J + \frac{1}{2}$	
C_{2h}	${}^1\Gamma_5$	$I = 1, \mu = \frac{1}{2}$	$\frac{1}{2} (1+I) \left(J + \frac{1}{2} \right)$	C_{2h}
	${}^1\Gamma_6$	1, $-\frac{1}{2}$	$\frac{1}{2} (1+I) \left(J + \frac{1}{2} \right)$	
	${}^1\Gamma_7$	-1, $\frac{1}{2}$	$\frac{1}{2} (1-I) \left(J + \frac{1}{2} \right)$	
	${}^1\Gamma_8$	-1, $-\frac{1}{2}$	$\frac{1}{2} (1-I) \left(J + \frac{1}{2} \right)$	
C_{2v}	${}^2\Gamma_5$	$\mu = \pm \frac{1}{2}$	$J + \frac{1}{2}$	C_2
D_2	${}^2\Gamma_5$	$\mu = \pm \frac{1}{2}$	$J + \frac{1}{2}$	C_2
D_{2h}	${}^2\Gamma_9$	$I = 1, \mu = \pm \frac{1}{2}$	$\frac{1}{2} (1+I) \left(J + \frac{1}{2} \right)$	C_{2h}
	${}^2\Gamma_{10}$	-1, $\pm \frac{1}{2}$	$\frac{1}{2} (1-I) \left(J + \frac{1}{2} \right)$	
C_3	${}^1\Gamma_5$	$\mu = \frac{1}{2}$	$1 + \left[\frac{J + \frac{1}{2}}{3} \right] + \left[\frac{J - \frac{1}{2}}{3} \right]$	C_3
	${}^1\Gamma_6$	$-\frac{1}{2}$	$1 + \left[\frac{J + \frac{1}{2}}{3} \right] + \left[\frac{J - \frac{1}{2}}{3} \right]$	
	${}^1\Gamma_4$	$\frac{3}{2}$	$1 + \left[\frac{J + \frac{3}{2}}{3} \right] + \left[\frac{J - \frac{3}{2}}{3} \right]$	

Tabelle 3 (Fortsetzung)

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
C_{3i}	${}^1\Gamma_8$	$\mu_I = \frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right] \right)$	C_{3i}
	${}^1\Gamma_9$	$-\frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right] \right)$	
	${}^1\Gamma_7$	$\frac{3}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	
	${}^1\Gamma_{10}$	0	$\frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	
	${}^1\Gamma_{11}$	1	$\frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right] \right)$	
	${}^1\Gamma_{12}$	-1	$\frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right] \right)$	
C_{3v}	${}^1\Gamma_4$	$S = i, \mu = \frac{3}{2}$	$\frac{1}{2} \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	C_3
	${}^1\Gamma_5$	$-i, \frac{3}{2}$	$\frac{1}{2} \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	
	${}^2\Gamma_6$	$\pm \frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right]$	
D_3	${}^1\Gamma_4$	$\nu = \frac{1}{2}, \mu = \frac{3}{2}$	$\frac{1}{2} \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	C_3
	${}^1\Gamma_5$	$-\frac{1}{2}, \frac{3}{2}$	$\frac{1}{2} \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	
	${}^2\Gamma_6$	$\pm \frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right]$	
D_{3d}	${}^1\Gamma_7$	$I = 1, \nu = \frac{1}{2}, \mu = \frac{3}{2}$	$\frac{1}{4} (1 + I) \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	C_3
	${}^1\Gamma_8$	$1, -\frac{1}{2}, \frac{3}{2}$	$\frac{1}{4} (1 + I) \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	
	${}^2\Gamma_9$	$1, \pm \frac{1}{2}$	$\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right] \right)$	
	${}^1\Gamma_{10}$	$-1, \frac{1}{2}, \frac{3}{2}$	$\frac{1}{4} (1 - I) \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	
	${}^1\Gamma_{11}$	$-1, -\frac{1}{2}, \frac{3}{2}$	$\frac{1}{4} (1 - I) \left(1 + \left[\frac{J+\frac{3}{2}}{3} \right] + \left[\frac{J-\frac{3}{2}}{3} \right] \right)$	

Tabelle 3 (Fortsetzung)

1 Sym- me- tric- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^2\Gamma_{12}$	$I = -1, \quad \mu = \pm \frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{3} \right] + \left[\frac{J-\frac{1}{2}}{3} \right] \right)$	C_3
C_4	${}^1\Gamma_5$	$\mu = \frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right]$	C_4
	${}^1\Gamma_6$	$-\frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right]$	
	${}^1\Gamma_7$	$\frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right]$	
	${}^1\Gamma_8$	$-\frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right]$	
S_4	${}^1\Gamma_5$	$\mu_I = \frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	S_4
	${}^1\Gamma_6$	$-\frac{1}{2}$	$+\frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
	${}^1\Gamma_7$	$\frac{3}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	
	${}^1\Gamma_8$	$-\frac{3}{2}$	$+\frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
C_{4v}	${}^1\Gamma_9$	$I = 1, \quad \mu = \frac{1}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	S_4
	${}^1\Gamma_{10}$	$1, \quad -\frac{1}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	
	${}^1\Gamma_{11}$	$1, \quad \frac{3}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
	${}^1\Gamma_{12}$	$1, \quad -\frac{3}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	

Tabelle 3 (Fortsetzung)

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^1\Gamma_{13}$	$I = -1, \quad \mu = \frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	C_{4h}
	${}^1\Gamma_{14}$	$-1, \quad -\frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	
	${}^1\Gamma_{15}$	$-1, \quad \frac{3}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
	${}^1\Gamma_{16}$	$-1, \quad -\frac{3}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
C_{4v}	${}^2\Gamma_6$	$\mu = \pm \frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right]$	C_4
	${}^2\Gamma_7$	$\pm \frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right]$	
D_4	wie C_{4v}	wie C_{4v}	wie C_{4v}	C_4
D_{2d}	${}^2\Gamma_6$	$\mu_I = \pm \frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	S_4
			$+ \frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
	${}^2\Gamma_7$	$\pm \frac{3}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
			$+ \frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	
D_{4h}	${}^2\Gamma_{11}$	$I = 1, \quad \mu = \pm \frac{1}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	C_{4h}
	${}^2\Gamma_{12}$	$1, \quad \pm \frac{3}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
	${}^2\Gamma_{13}$	$-1, \quad \pm \frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{4} \right] + \left[\frac{J-\frac{1}{2}}{4} \right] \right)$	
	${}^2\Gamma_{14}$	$-1, \quad \pm \frac{3}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{3}{2}}{4} \right] + \left[\frac{J-\frac{3}{2}}{4} \right] \right)$	
C_6	${}^1\Gamma_7$	$\mu = \frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right]$	
	${}^1\Gamma_8$	$-\frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right]$	

Tabelle 3 (Fortsetzung)

1 Sym- me- trie- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^1\Gamma_9$	$\mu = \frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right]$	C_6
	${}^1\Gamma_{10}$	$-\frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right]$	
	${}^1\Gamma_{11}$	$\frac{5}{2}$	$1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right]$	
	${}^1\Gamma_{12}$	$-\frac{5}{2}$	$1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right]$	
C_{3h}	${}^1\Gamma_7$	$\mu_I = \frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	C_{3h}
			$+ \frac{1}{2} \left(1 - (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
	${}^1\Gamma_8$	$-\frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
			$+ \frac{1}{2} \left(1 - (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
	${}^1\Gamma_9$	$\frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right]$	
	${}^1\Gamma_{10}$	$-\frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right]$	
	${}^1\Gamma_{11}$	$\frac{5}{2}$	$\frac{1}{2} \left(1 + (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
			$+ \frac{1}{2} \left(1 - (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
	${}^1\Gamma_{12}$	$-\frac{5}{2}$	$\frac{1}{2} \left(1 + (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
			$+ \frac{1}{2} \left(1 - (-1)^k \sum l_k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
C_{3h}	${}^1\Gamma_{13}$	$I = 1, \quad \mu = \frac{1}{2}$	$\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	C_{3h}
	${}^1\Gamma_{14}$	$1, \quad -\frac{1}{2}$	$\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
	${}^1\Gamma_{15}$	$1, \quad \frac{3}{2}$	$\frac{1}{2} (1 + I) \left(1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right] \right)$	

Tabelle 3 (Fortsetzung)

1 Sym- me- trie- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
	${}^1\Gamma_{16}$	$I = 1, \quad \mu = -\frac{3}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right] \right)$	C_{6A}
	${}^1\Gamma_{17}$	$1, \quad \frac{5}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
	${}^1\Gamma_{18}$	$1, \quad -\frac{5}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
	${}^1\Gamma_{19}$	$-1, \quad \frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
	${}^1\Gamma_{20}$	$-1, \quad -\frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
	${}^1\Gamma_{21}$	$-1, \quad \frac{3}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right] \right)$	
	${}^1\Gamma_{22}$	$-1, \quad -\frac{3}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right] \right)$	
	${}^1\Gamma_{23}$	$-1, \quad \frac{5}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
	${}^1\Gamma_{24}$	$-1, \quad -\frac{5}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
C_{6v}	${}^2\Gamma_7$	$\mu = \pm \frac{1}{2}$	$1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right]$	C_6
	${}^2\Gamma_9$	$\pm \frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right]$	
	${}^2\Gamma_8$	$\pm \frac{5}{2}$	$1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right]$	
D_6 D_{3h}	wie C_{6v}	wie C_{6v}	wie C_{6v}	C_6
	${}^2\Gamma_7$	$\mu_1 = \pm \frac{1}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right) + \frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	C_{3A}
	${}^2\Gamma_9$	$\pm \frac{3}{2}$	$1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right]$	
	${}^2\Gamma_8$	$\pm \frac{5}{2}$	$\frac{1}{2} \left(1 + (-1)^k \right) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right) + \frac{1}{2} \left(1 - (-1)^k \right) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	

Tabelle 3 (Fortsetzung)

1 Sym- me- trie- klasse	2 nach d. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
D_{6h}	${}^2\Gamma_{13}$	$I = 1, \quad \mu = \pm \frac{1}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	C_{6h}
	${}^2\Gamma_{15}$	$1, \quad \pm \frac{3}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right] \right)$	
	${}^2\Gamma_{14}$	$1, \quad \pm \frac{5}{2}$	$\frac{1}{2}(1+I) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	
	${}^2\Gamma_{16}$	$-1, \quad \pm \frac{1}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{1}{2}}{6} \right] + \left[\frac{J-\frac{1}{2}}{6} \right] \right)$	
	${}^2\Gamma_{18}$	$-1, \quad \pm \frac{3}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{3}{2}}{6} \right] + \left[\frac{J-\frac{3}{2}}{6} \right] \right)$	
	${}^2\Gamma_{17}$	$-1, \quad \pm \frac{5}{2}$	$\frac{1}{2}(1-I) \left(1 + \left[\frac{J+\frac{5}{2}}{6} \right] + \left[\frac{J-\frac{5}{2}}{6} \right] \right)$	

[Errata]
Correction of the Work of K. H. Hellwege:
Electron Terms and Splitting of Atoms in Crystals
I. Term Splitting and Electric Dipole Radiation
Ann. Physik 6, 95 (1948)

Unfortunately, some errors slipped into the stated work, which are to be corrected as follows.

In Tables 2, the sections from pp. 110-111 and 113-114 for the symmetry classes D_{2d} and D_{3h} are correctly:
[See tables from Errata, following.]

[Text near the bottom of page 358.]

Further, on p. 113, the last row from D_6 is correctly:
[See tables from Errata, following.]

[All following errata have already been exercised upon the English text.]

On page 99 beneath Eq. (14) the clause "where M is an arbitrary whole number" is to be stricken.

On page 104 in the line of Eq. (43): $\mu = 3/2$ instead of $1/2$ and in the line after Eq. (43): Z' instead of Z_1 .

On page 105 last line: Z' instead of Z_1 .

Göttingen, II. Physical Institute
(Arrived at the editor's office 20 January, 1949.)

Berichtigung zur Arbeit von K. H. Hellwege:
Elektronenterme und Strahlung von Atomen in Kristallen
I. Termaufspaltung und elektrische Dipolstrahlung
Ann. Physik (6) 4, 95 (1948)

In die genannte Arbeit haben sich leider einige Schreibfehler eingeschlichen, die wie folgt zu korrigieren sind.

In Tabelle 2 muß es S. 110/111 und S. 113/114 für die Symmetrieklassen D_{2d} und D_{3h} richtig so heißen:

1 Sym- me- trie- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
D_{2d}	${}^1\Gamma_1$	$S = 1, \mu_I = 0$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $+ \frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^1\Gamma_2$	$-1, \quad 0$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $- \frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^1\Gamma_3$	$1, \quad 2$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^1\Gamma_4$	$-1, \quad 2$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+2}{4} \right] + \left[\frac{J-2}{4} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{4} \right] \right)$ $- \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^2\Gamma_6$	± 1	$1 + \left[\frac{J+1}{4} \right] + \left[\frac{J-1}{4} \right]$	S_4

1 Sym- me- trie- klasse	2 nachd. Grup- penth.	3 Termfamilien nach Kristallquantenzahlen	4 Anzahl der Terme in jeder Familie bei gegebenem J	5 Voll- ständ. Aufsp. in
D_{3h}	${}^1\Gamma_1$	$S = 1, \mu_I = 0$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^1\Gamma_2$	$-1, 0$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $- \frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^1\Gamma_3$	$1, 3$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^1\Gamma_4$	$-1, 3$	$\frac{1}{4} \left(1 + (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+3}{6} \right] + \left[\frac{J-3}{6} \right] \right)$ $+ \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) \left(1 + 2 \left[\frac{J}{6} \right] \right)$ $- \frac{1}{4} \left(1 - (-1)^{\sum l_k} \right) (-1)^{\sum (l_k - j_k)}$	
	${}^2\Gamma_6$	± 1	$\frac{1}{2} \left(1 + (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$ $+ \frac{1}{2} \left(1 - (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$	
	${}^2\Gamma_5$	± 2	$\frac{1}{2} \left(1 + (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right] \right)$ $+ \frac{1}{2} \left(1 - (-1)^{\sum l_k} \right) \left(1 + \left[\frac{J+1}{6} \right] + \left[\frac{J-1}{6} \right] \right)$	C_{3h}

Ferner heißt auf S. 113 die letzte Zeile von D_6 richtig:

${}^2\Gamma_6$	± 2	$1 + \left[\frac{J+2}{6} \right] + \left[\frac{J-2}{6} \right]$	C_6
----------------	---------	---	-------

Auf S. 99 ist hinter Gl. (14) der Satz „wobei m eine beliebige ganze Zahl ist“ zu streichen.

Auf S. 104 in der Zeile vor Gl. (43): $\mu = 3/2$ statt $\mu = 1/2$ und in der Zeile nach Gl. (43): Z' statt Z_1 .

Auf S. 105 letzte Zeile: Z' statt Z_1 .

Göttingen, II. Physikalisches Institut.

(Bei der Redaktion eingegangen am 20. Januar 1949.)

Verantwortlich
für die Schriftleitung: Prof. Dr. Friedrich Möglich, Berlin-Buch, Lindenberger Weg 74
für den Verlag: Hofrat Dr. Arthur Meiner, Leipzig C 1, Salomonstraße 18 B
für den Anzeigenteil: Kurt Klimmer (Arbeitsgemeinschaft medizinischer
Verlage G. m. b. H.), Berlin C 2, Neue Grünstraße 18, Fernruf: 42 30 97
(1) Paul Dünhaupt, Köthen L 22

Lizenz-Nr. 410 / G. N. 446